

## AS Level Further Mathematics A

### Y531 Pure Core

#### Sample Question Paper

Version 2

## Date – Morning/Afternoon

Time allowed: 1 hour 15 minutes

#### You must have:

- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

#### You may use:

- a scientific or graphical calculator



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### INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

### INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

**1 In this question you must show detailed reasoning.**

The equation  $x^2 + 2x + 5 = 0$  has roots  $\alpha$  and  $\beta$ . The equation  $x^2 + px + q = 0$  has roots  $\alpha^2$  and  $\beta^2$ . Find the values of  $p$  and  $q$ . [3]

**2 In this question you must show detailed reasoning.**

Given that  $z_1 = 3 + 2i$  and  $z_2 = -1 - i$ , find the following, giving each in the form  $a + bi$ .

(i)  $z_1^* z_2$  [2]

(ii)  $\frac{z_1 + 2z_2}{z_2}$  [2]

**3 (i) You are given two matrices,  $\mathbf{A}$  and  $\mathbf{B}$ , where**

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}.$$

Show that  $\mathbf{AB} = m\mathbf{I}$ , where  $m$  is a constant to be determined. [2]

**(ii) You are given two matrices,  $\mathbf{C}$  and  $\mathbf{D}$ , where**

$$\mathbf{C} = \begin{pmatrix} 2 & 1 & 5 \\ 1 & 1 & 3 \\ -1 & 2 & 2 \end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix} -4 & 8 & -2 \\ -5 & 9 & -1 \\ 3 & -5 & 1 \end{pmatrix}.$$

Show that  $\mathbf{C}^{-1} = k\mathbf{D}$  where  $k$  is a constant to be determined. [2]

**(iii) The matrices  $\mathbf{E}$  and  $\mathbf{F}$  are given by  $\mathbf{E} = \begin{pmatrix} k & k^2 \\ 3 & 0 \end{pmatrix}$  and  $\mathbf{F} = \begin{pmatrix} 2 \\ k \end{pmatrix}$  where  $k$  is a constant.**

Determine any matrix  $\mathbf{F}$  for which  $\mathbf{EF} = \begin{pmatrix} -2k \\ 6 \end{pmatrix}$ . [5]

**4 Draw the region of the Argand diagram for which  $|z - 3 - 4i| \leq 5$  and  $|z| \leq |z - 2|$ . [4]**

5 The matrix  $\mathbf{M}$  is given by  $\mathbf{M} = \begin{pmatrix} -\frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix}$ .

(i) The diagram in the Printed Answer Booklet shows the unit square  $OABC$ . The image of the unit square under the transformation represented by  $\mathbf{M}$  is  $OA'B'C'$ . Draw and clearly label  $OA'B'C'$ . [3]

(ii) Find the equation of the line of invariant points of this transformation. [3]

(iii) (a) Find the determinant of  $\mathbf{M}$ . [1]

(b) Describe briefly how this value relates to the transformation represented by  $\mathbf{M}$ . [2]

6 At the beginning of the year John had a total of £2000 in three different accounts. He has twice as much money in the current account as in the savings account.

- The current account has an interest rate of 2.5% per annum.
- The savings account has an interest rate of 3.7% per annum.
- The supersaver account has an interest rate of 4.9% per annum.

John has predicted that he will earn a total interest of £92 by the end of the year.

(i) Model this situation as a matrix equation. [2]

(ii) Find the amount that John had in each account at the beginning of the year. [2]

(iii) In fact, the interest John will receive is £92 to the nearest pound. Explain how this affects the calculations. [2]

7 **In this question you must show detailed reasoning.**

It is given that  $f(z) = z^3 - 13z^2 + 65z - 125$ .

The points representing the three roots of the equation  $f(z) = 0$  are plotted on an Argand diagram.

Show that these points lie on the circle  $|z| = k$ , where  $k$  is a real number to be determined. [9]

8 Prove that  $n! > 2^n$  for  $n \geq 4$ . [5]

- 9 (i) Find the value of  $k$  such that  $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$  and  $\begin{pmatrix} -2 \\ 3 \\ k \end{pmatrix}$  are perpendicular. [2]

Two lines have equations  $l_1 : \mathbf{r} = \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$  and  $l_2 : \mathbf{r} = \begin{pmatrix} 6 \\ 5 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ .

- (ii) Find the point of intersection of  $l_1$  and  $l_2$ . [4]

- (iii) The vector  $\begin{pmatrix} 1 \\ a \\ b \end{pmatrix}$  is perpendicular to the lines  $l_1$  and  $l_2$ .

Find the values of  $a$  and  $b$ .

[5]

**END OF QUESTION PAPER**

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